Sapienza
Università di Roma

Can we think timelessly about causation?

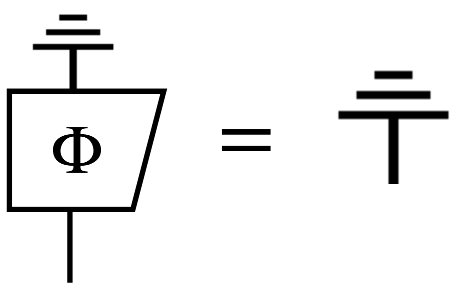
Andrea Di Biagio

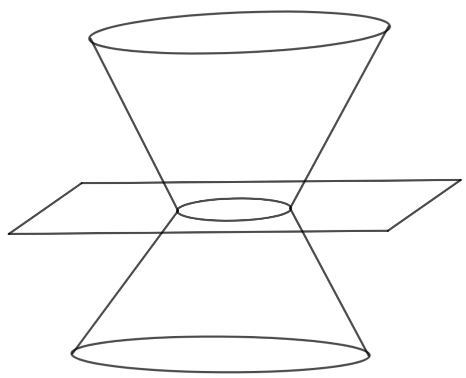
4 Dec 2020

Qubits and Spacetime
Interview/Talk

Starting tension

No signalling from the future: An OPT is **causal** if the probabilities of an operation do not depend on the choice of any *later* operation.





Relativistic Causality: A change in the initial data in a region *S*, does not produce any change in the regions outside the causal *past and future* of *S*.

Reconstructions

Lucien Hardy, "Quantum Theory From Five Reasonable Axioms," (2001), arXiv:quant-ph/0101012.

Borivoje Dakic and Časlav Brukner, "Quantum theory and beyond: Is entanglement special?" (2009), arXiv:0911.0695 [quant-ph].

Lluís Masanes and Markus P. Müller, "A derivation of quantum theory from physical requirements," New Journal of Physics **13**, 063001 (2011).

G. Chiribella, G. M. D'Ariano, and P. Perinotti, "Informational derivation of Quantum Theory," Physical Review A **84**, 012311 (2011), arXiv:1011.6451.

Lucien Hardy, "Reconstructing quantum theory," (2013), arXiv:1303.1538 [gr-qc, physics:hep-th, physics:quant-ph].

Philipp A. Höhn, "Toolbox for reconstructing quantum theory from rules on information acquisition," Quantum **1**, 38 (2017), arXiv:1412.8323.

Philipp A. Höhn and Christopher Wever, "Quantum theory from questions," Physical Review A **95**, 012102 (2017), arXiv:1511.01130.

John H. Selby, Carlo Maria Scandolo, and Bob Coecke, "Reconstructing quantum theory from diagrammatic postulates," arXiv:1802.00367 [quant-ph] (2018), arXiv:1802.00367 [quant-ph].

Ding Jia, "Quantum from principles without assuming definite causal structure," Physical Review A **98**, 032112 (2018), arXiv:1808.00898.

Robert Oeckl, "A local and operational framework for the foundations of physics," Advances in Theoretical and Mathematical Physics **23**, 437–592 (2019), arXiv:1610.09052.

Starting tension

Does quantum mechanics imply time orientation?

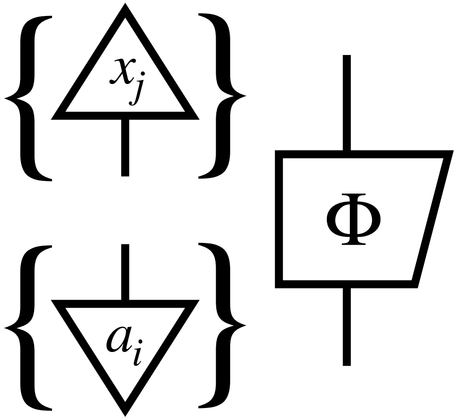
Plan

- Quantum Information and the arrow of time
- Towards time-symmetric causation
- Next steps

Plan

- **Quantum Information and the arrow of time**
- Towards time-symmetric causation
- Next steps

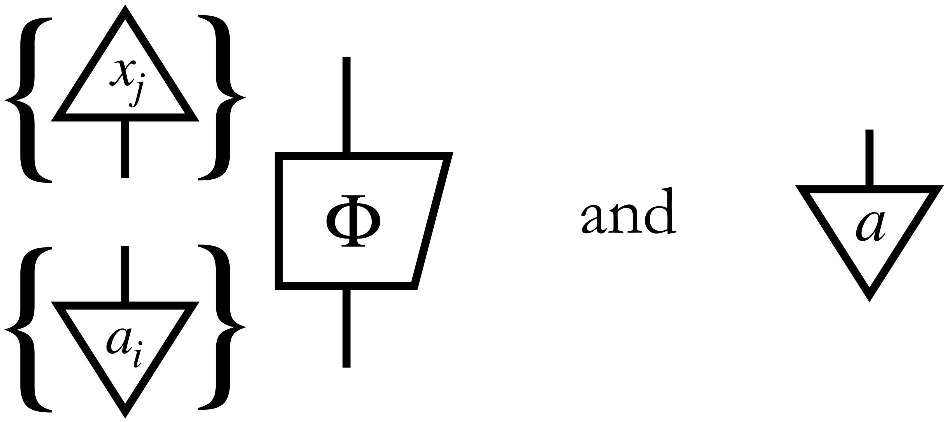
Two Games



Prediction vs Postdiction

Prediction: Given a preparation, a test and the result of the preparation, calculate the probabilities of the outcomes of the test.

Given

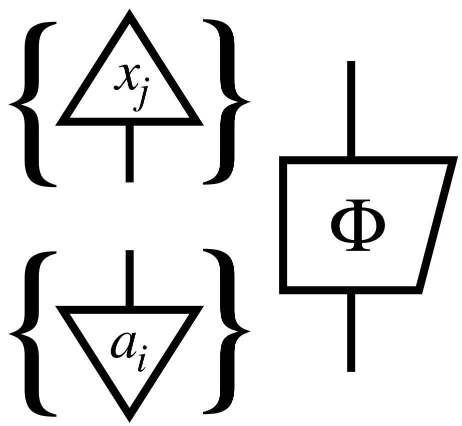


find $$P_{pre}(x_j \,|\, a, \Phi)$$

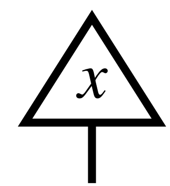
Prediction vs Postdiction

Postdiction: Given a preparation, a test and the result of the *test*, calculate the probabilities of the outcomes of the *preparation*.

Given

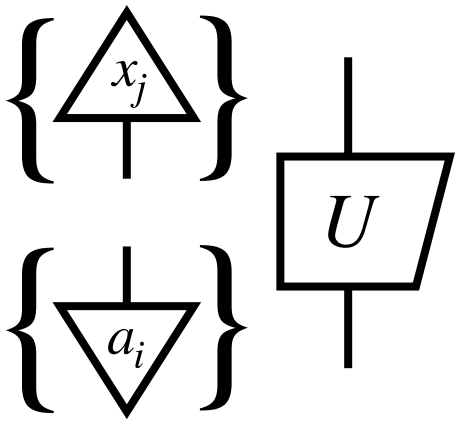


and



find $$P_{post}(a_i \,|\, x, \Phi)$$

Closed Systems

Given 

Born rule

$$P_{pre}(x\,|\,a, U) = |\langle x\,|\,U\,|\,a\rangle|^2$$

Bayes' theorem

$$P_{post}(a\,|\,x, U) = \frac{P_{pre}(x\,|\,a, U)P(a)}{P(x)}$$

What are $P(a)$ and $P(x)$?

We are doing inference using the Born rule.

$P(a)$ and $P(x)$ are *a priori* probabilities.

Prior $P(a) = \frac{1}{d}$

Data $P(x) = \sum_{i=1}^{d} P_{pre}(x \mid a_i, U) P(a_i) = \sum_{i=1}^{d} \left| \langle x \mid U \mid a_i \rangle \right|^2 \cdot \frac{1}{d} = \frac{1}{d}$

$$P_{post}(a \mid x, U) = \frac{P_{pre}(x \mid a, U) P(a)}{P(x)} = P_{pre}(x \mid a, U)$$

We are doing inference using the Born rule.

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$$P_{post}(a \mid x, \Phi) = |\langle x \mid U \mid a \rangle|^2 = P_{pre}(x \mid a, U)$$

Time agnostic probabilities

A process Φ is **inference symmetric** if:

$$P_{pre}(x_j | a_i, \Phi) = P_{post}(a_i | x_j, \Phi)$$

for any choice of bases.

Closed quantum systems are inference symmetric.

Time-Reversal

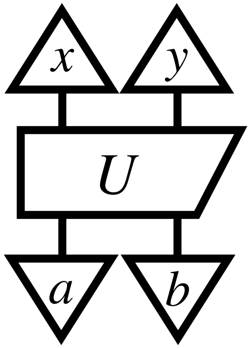
Passive: Describe physical events in reversed order.

Active: Find a process that undoes the original process.

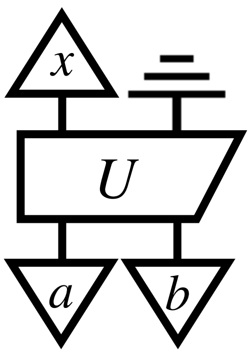
$P_{pre}(x\,|\,a, U)$ ⟵ **Passive** ⟶ $P_{post}(a\,|\,x, U)$

Active ↕ $|\langle x|U|a\rangle|^2$ ↕ **Active**

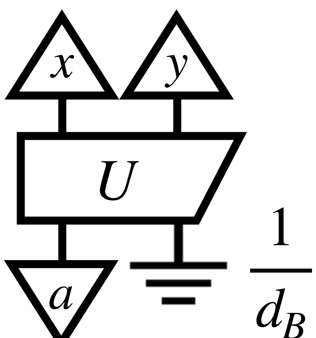
$P_{pre}(a\,|\,x, U^{\dagger})$ ⟵ **Passive** ⟶ $P_{post}(x\,|\,a, U^{\dagger})$



$$P_{pre}(xy\,|\,ab, U) = P_{post}(ab\,|\,xy, U)$$

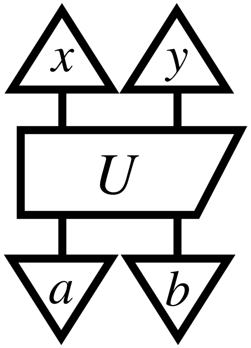


$$P_{pre}(x\,|\,ab, U) = \sum_{i=1}^{d_Y} P_{pre}(xy_i\,|\,ab, U)$$

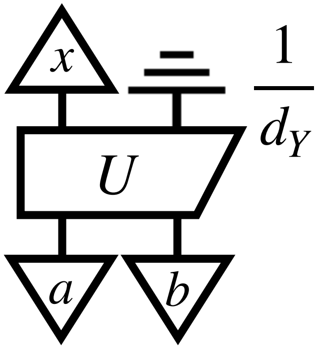


$$P_{pre}(xy\,|\,a, U) = \frac{1}{d_B}\sum_{i=1}^{d_B} P_{pre}(xy\,|\,ab_i, U)$$

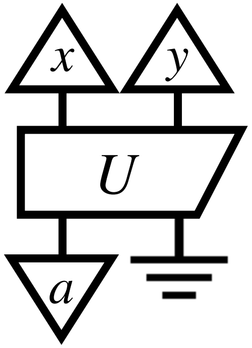
Open Systems



$$P_{pre}(xy\,|\,ab, U) = P_{post}(ab\,|\,xy, U)$$



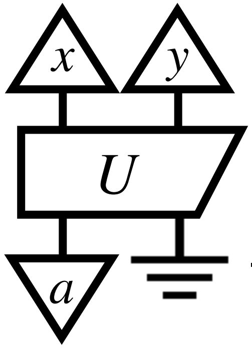
$$P_{post}(ab\,|\,x, U) = \frac{1}{d_Y}\sum_{i=1}^{d_Y} P_{post}(ab\,|\,xy_i, U)$$



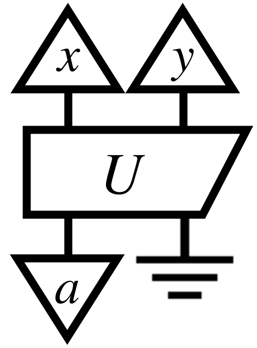
$$P_{post}(a\,|\,xy, U) = \sum_{i=1}^{d_B} P_{post}(ab_i\,|\,xy, U)$$

Direction of inference

$P_{pre}(xy\,|\,a, U) =$

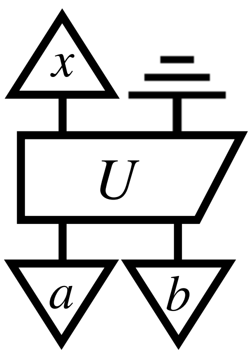


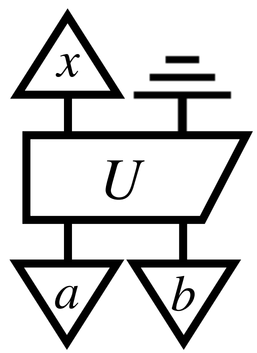
$\frac{1}{d_B}$



$= P_{post}(a\,|\,xy, U)$

$P_{pre}(x\,|\,ab, U) =$





$\frac{1}{d_Y}$

$= P_{post}(ab\,|\,x, U)$

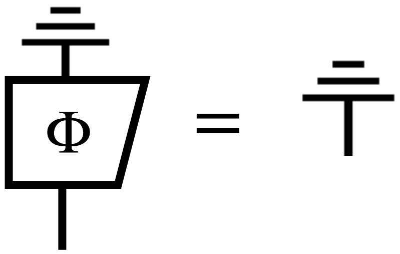
Direction of inference

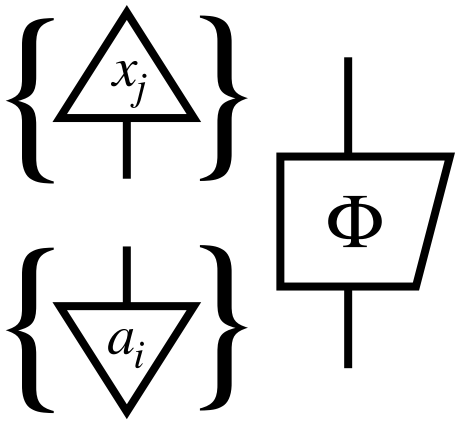
$$P_{pre}(x\,|\,ab, U) = d_Y P_{post}(ab\,|\,x, U)$$

$$P_{pre}(xy\,|\,a, U) = \frac{1}{d_B} P_{post}(a\,|\,xy, U)$$

Inference symmetry broken in the simplest way

A quantum channel is represented by a CPTP map.



Given 

Generalised Born rule

$$P_{pre}(x \mid a, \Phi) = \mathrm{tr}\, |x\rangle\langle x| \, \Phi[\, |a\rangle\langle a| \,]$$

Bayes' theorem

$$P_{post}(a \mid x, \Phi) = \frac{P_{pre}(x \mid a, \Phi) P(a)}{P(x)}$$

Channels

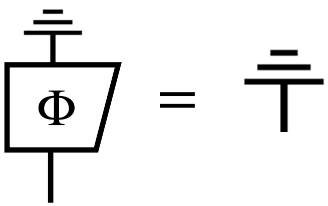
Prior $$P(a) = \frac{1}{d_A}$$

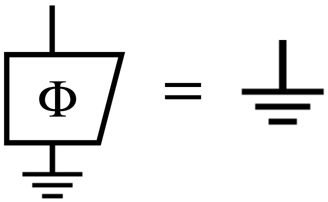
Data $$P(x) = \sum_{i=1}^{d_A} \frac{1}{d_A} P_{pre}(x \,|\, a_i, \Phi) = \frac{1}{d_A} \mathrm{tr}\, |x\rangle\langle x|\, \Phi[\mathbb{1}_A]$$

$$P_{post}(a \,|\, x, \Phi) = \frac{\mathrm{tr}\, |x\rangle\langle x|\, \Phi[\,|a\rangle\langle a|\,]}{\mathrm{tr}\, |x\rangle\langle x|\, \Phi[\mathbb{1}_A]} = \frac{P_{pre}(x \,|\, a, \Phi)}{\mathrm{tr}\, |x\rangle\langle x|\, \Phi[\mathbb{1}_A]}$$

Inference Symmetric Channels

$$P_{post}(a\,|\,x,\Phi) = \frac{P_{pre}(x\,|\,a,\Phi)}{\mathrm{tr}\,|x\rangle\langle x|\,\Phi[\mathbb{I}_A]}$$

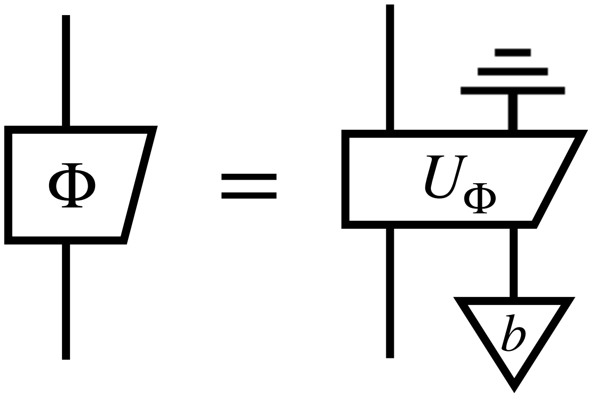




A channel is **inference symmetric** iff it is **bistochastic**.

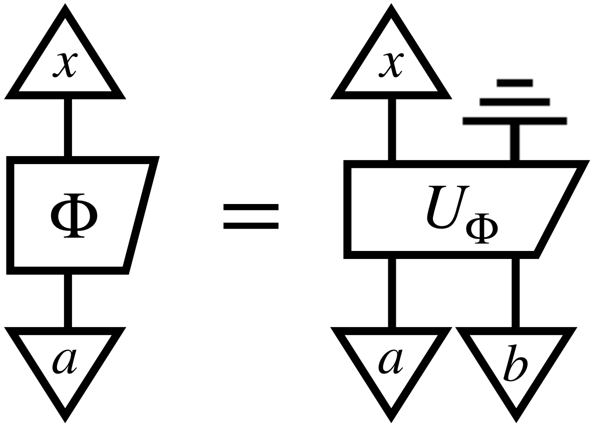
Purification

Any quantum channel can be understood in terms of a unitary interaction with an ancilla system.



This allows us to understand the inference asymmetry of the quantum channels.

$$P_{post}(x\,|\,a,\Phi) = P_{post}(a\,|\,xb, U_\Phi)$$

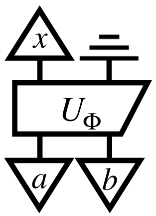


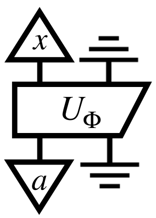
Purification

$$P_{post}(a|xb, U_\Phi) = \frac{P_{post}(ab|x, U_\Phi)}{P_{post}(b|x, U_\Phi)}$$

$$P(a|b) = \frac{P(ab)}{P(b)}$$

$$P_{post}(ab|x, U_\Phi) = \frac{1}{d_Y} P_{pre}(x|ab, U_\Phi)$$





$$P_{post}(b|x, U_\Phi) = \frac{d_A}{d_Y} P_{pre}(x|b, U_\Phi)$$

$$P_{post}(a|xb, U_\Phi) = \frac{P_{pre}(x|ab, U_\Phi)}{d_A P_{pre}(x|b, U_\Phi)}$$

Purification

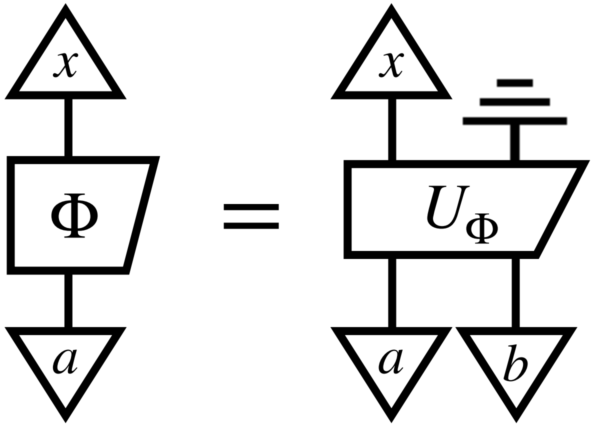
$$P_{post}(a\,|\,xb, U_\Phi) = \frac{P_{pre}(x\,|\,ab, U_\Phi)}{d_A P_{pre}(x\,|\,b, U_\Phi)}$$

$$P_{pre}(x\,|\,ab, U_\Phi) = P_{pre}(x\,|\,a, \Phi)$$

$$P_{pre}(x\,|\,b, U_\Phi) = \sum_{i=1}^{d_A} \frac{1}{d_A} P_{pre}(x\,|\,a_i b, U_\Phi) = \frac{1}{d_A} \sum_{i=1}^{d_A} P_{pre}(x\,|\,a_i, \Phi)$$

$$P_{post}(a\,|\,xb, U_\Phi) = \frac{P_{pre}(x\,|\,ab, U_\Phi)}{\mathrm{tr}\,|x\rangle\langle x|\,\Phi[\mathbb{I}_A]} = P_{post}(a\,|\,x, \Phi)$$

$$P_{post}(x\,|\,a,\Phi) = P_{post}(a\,|\,xb,U_\Phi)$$



The inference asymmetry of quantum channels is understood as an asymmetry in the boundary data.

Two Maxims

There exists a unique deterministic effect.

The choice of an operation does not affect the probabilities of the outcome of an earlier operation.

Maxim 1

There exists a unique deterministic effect.

Mathematically correct: the trace is the only CPTP map to the trivial space.

Physically correct: there is fundamental unpredictability in QM.

But not a difference between past and future: there is fundamental un*post*dictability in QM.

Maxim 1

When *predicting*, there exists a unique deterministic *effect*.

When *postdicting*, there exists a unique deterministic *state*.

Maxim 2

The choice of an operation does not affect the probabilities of the outcome of an earlier operation.

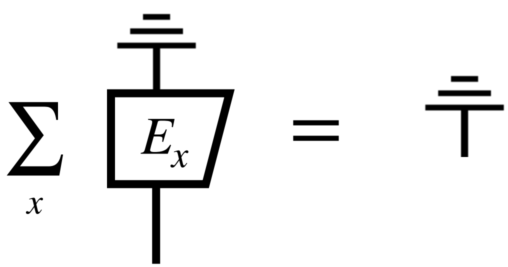
Mathematically correct: a consequence of conservation of probabilities.

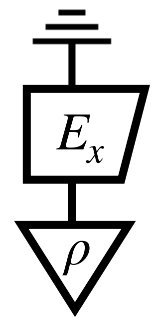
Physically correct: experimentally corroborated.

But not a difference between past and future:
difference between known and unknown

An operation is a set $E = \{E_x\}$ of CP maps such that $\sum_x E_x$ is a CPTP map.

The probability of outcome x is given by $P(x\,|\,\rho, E) = \text{tr}\ E_x[\rho]$.



$P(x\,|\,\rho, E) =$ 

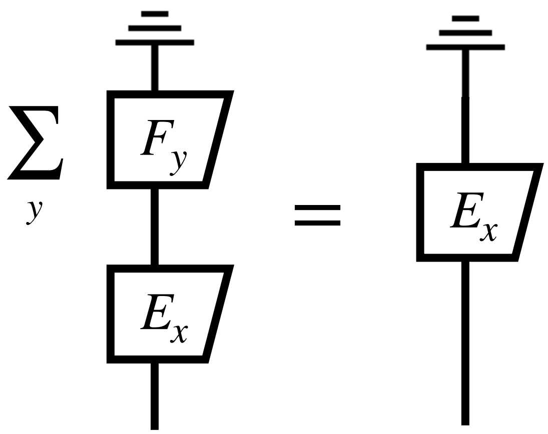
If two operations $\{E_x\}$ and $\{F_y\}$ are composed in sequence:

$$P(xy\,|\,\rho, F \circ E) = \text{tr}\ F_y\big[E_x[\rho]\big].$$

Maxim 2

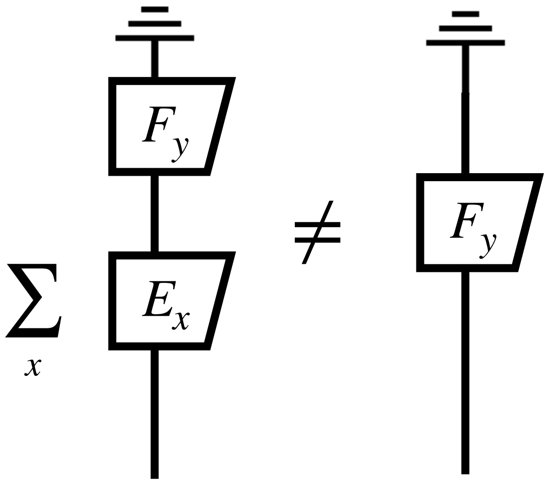
Then

$$P(x\,|\,\rho, F\circ E) = \sum_y \operatorname{tr}\, F_y\big[E_x[\rho]\big] = \operatorname{tr}\, E_x[\rho] = P(x\,|\,\rho, E)$$

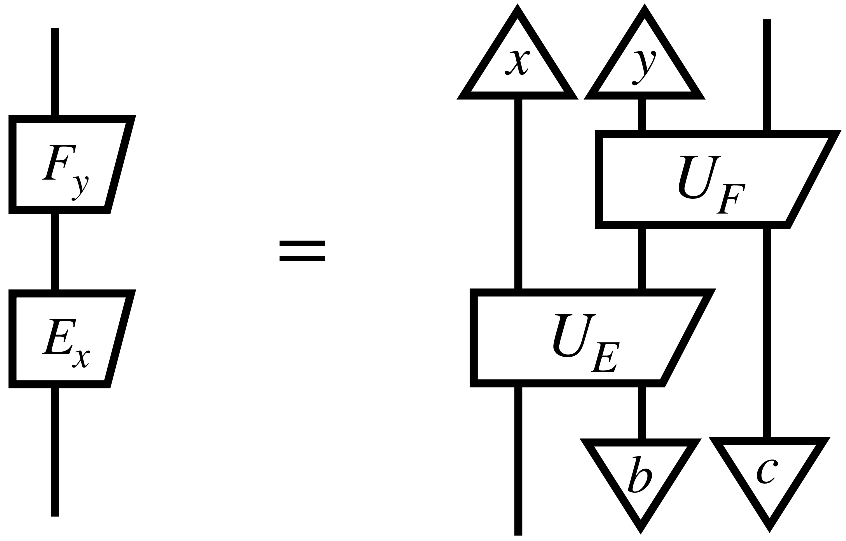


But clearly

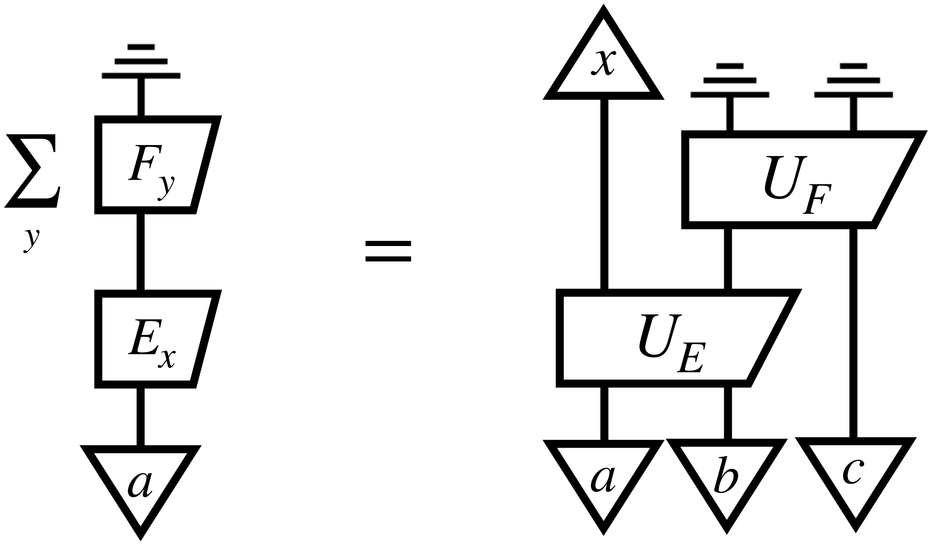
$$P_{pre}(y\,|\,\rho, F\circ E) = \sum_x \mathrm{tr}\; F_y\big[E_x[\rho]\big] = \mathrm{tr}\; F_y\big[E[\rho]\big]$$



Maxim 2

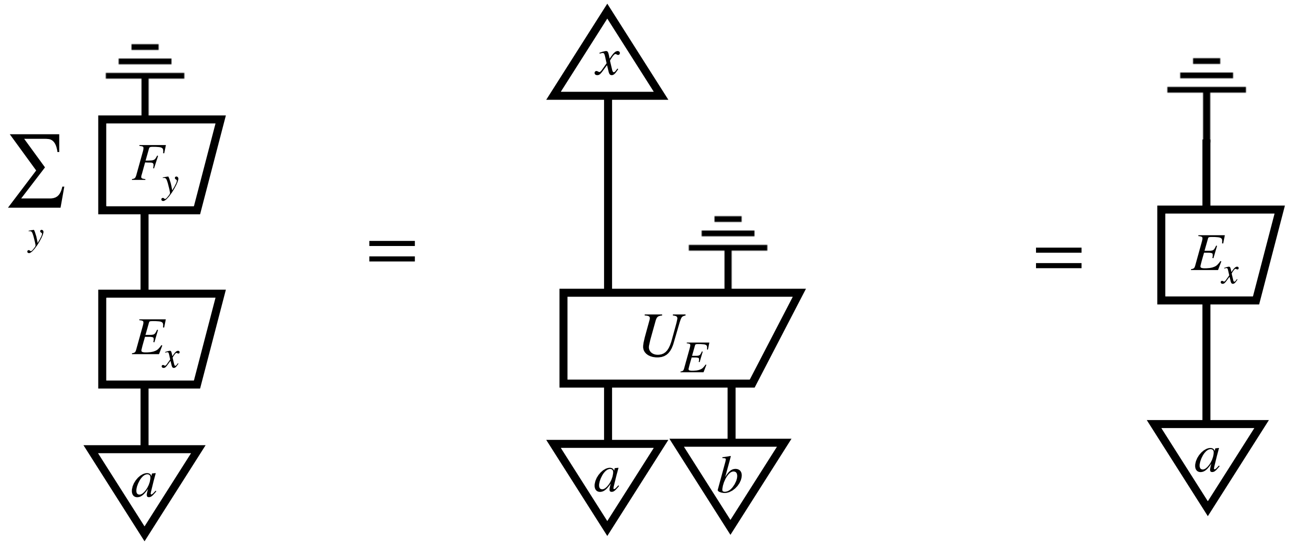


Maxim 2

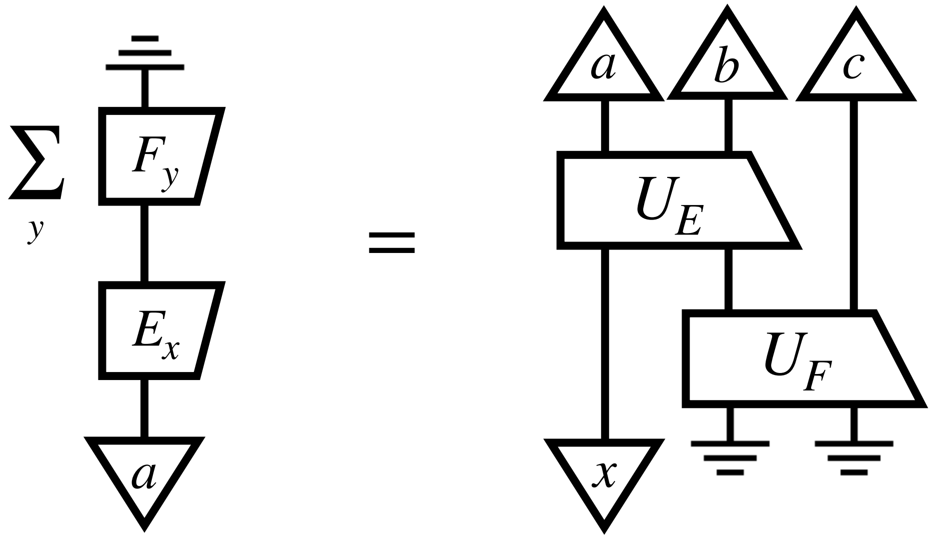


$$P(x\,|\,a, F\circ E) = P_{pre}(x\,|\,abc, U_F\circ U_E)$$

Maxim 2



$$P(x\,|\,a, F\circ E) = P_{pre}(x\,|\,abc, U_F\circ U_E) = P(x\,|\,a, E)$$



$$P(x\,|\,a, F \circ E) = P_{post}(x\,|\,abc, U_E^{\dagger} \circ U_F^{\dagger})$$

Why the asymmetry?

There are two asymmetric aspects:

- We are interested in prediction
- We consider time-asymmetric boundary conditions

Both can be understood in terms of thermodynamics:

- We remember the past, and not the future
- We make choices that affect the future, not the past

Ismael, *How physics makes us free*, Oxford University Press (2016)

Price, *Time's arrow & Archimedes' point*, Oxford University Press (1997)

Mlodinow and Brun, *Relation between the psychological and thermodynamic arrows of time*. Phys. Rev. E **89**, (2014)

Rovelli, *Agency in Physics*. arXiv:2007.05300 (2020)

Rovelli, *Memory and entropy*. arXiv:2003.06687 (2020)

arXiv

[Submitted on 12 Oct 2020] http://arxiv.org/abs/2010.05734

Quantum information and the arrow of time

Andrea Di Biagio, Pietro Donà, Carlo Rovelli

Plan

- Quantum Information and the arrow of time
- **Towards time-symmetric causation**
- Next steps

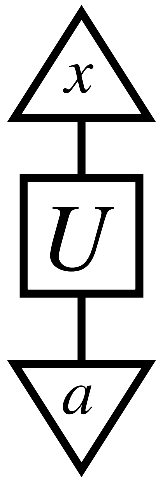
Take aways

Transition probabilities do not care about the direction of time.

There is a difference between known and unknown.

Diagrammatic calculus is useful for systemic thinking, very legible, and suited for "distributed" processes.

But... there is a strange mix between the "physical" and "inferential" aspect of the theory.



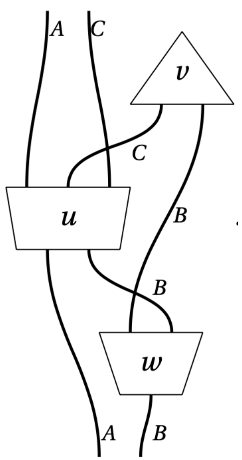
Also, all probabilities are implicitly *prediction* probabilities.

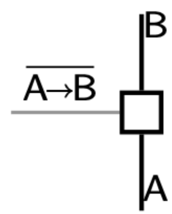
Unscrambling

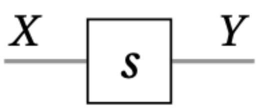
[Submitted on 7 Sep 2020 (v1), last revised 9 Sep 2020 (this version, v2)]

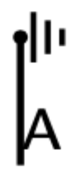
Unscrambling the omelette of causation and inference: The framework of causal-inferential theories

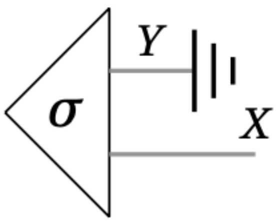
David Schmid, John H. Selby, Robert W. Spekkens

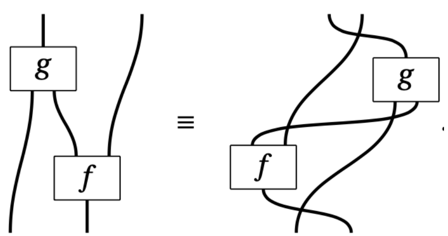


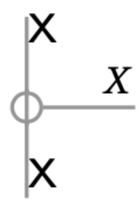


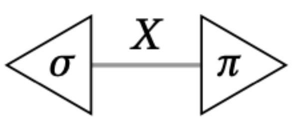




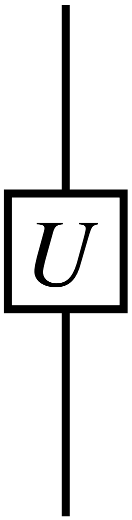




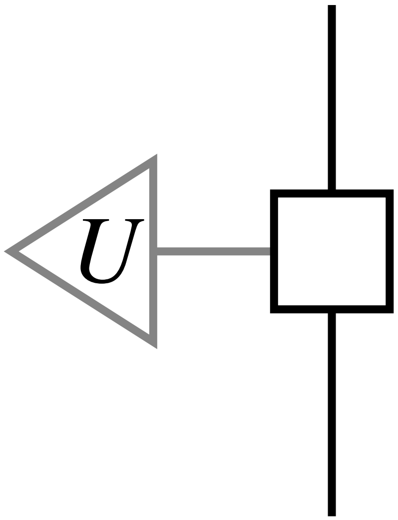




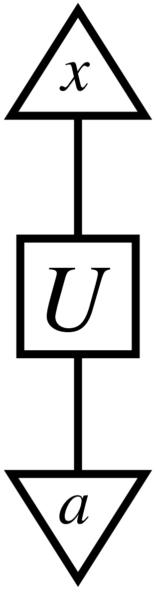
Unitary



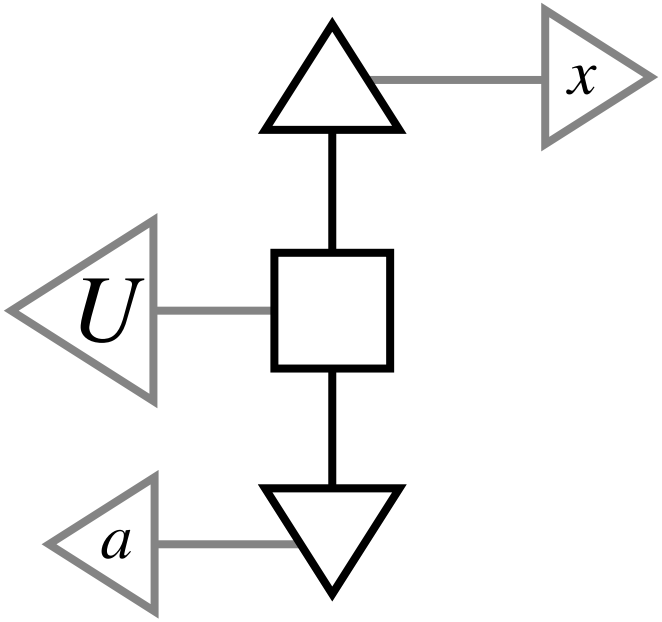
Unitary



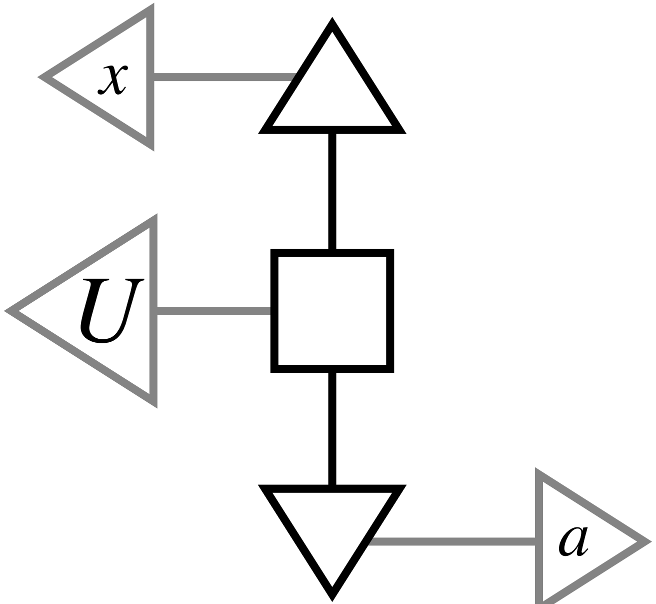
Prepare-Measure



Prediction

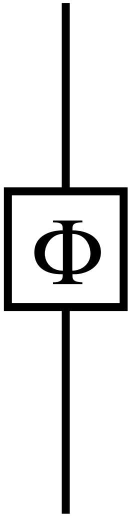


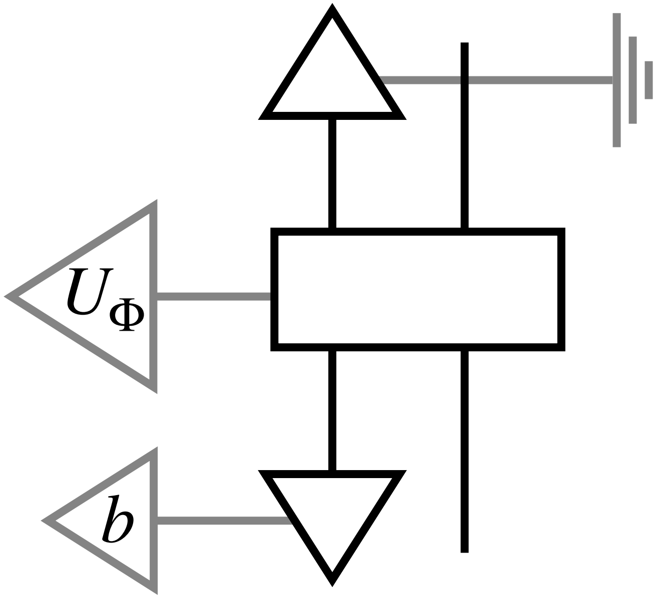
$$= P_{pre}(x \,|\, a, U)$$

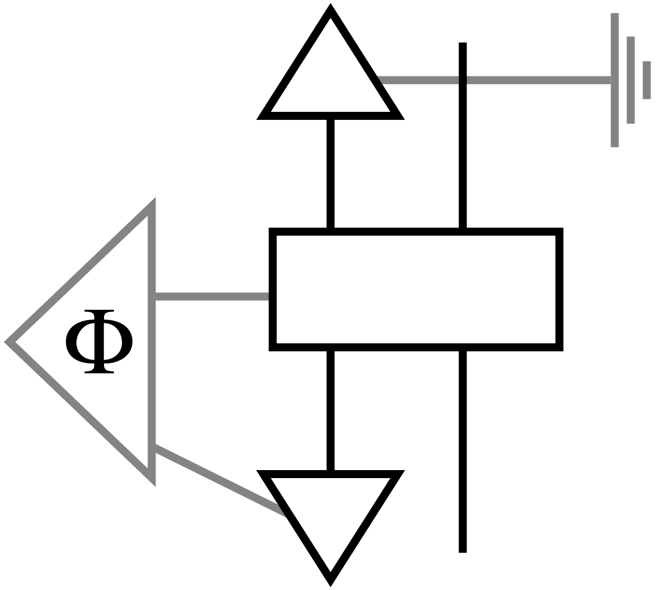


$$= P_{post}(a \mid x, U)$$

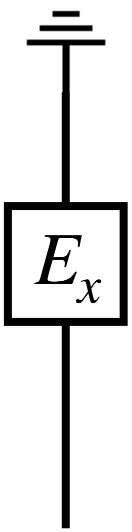
Channel



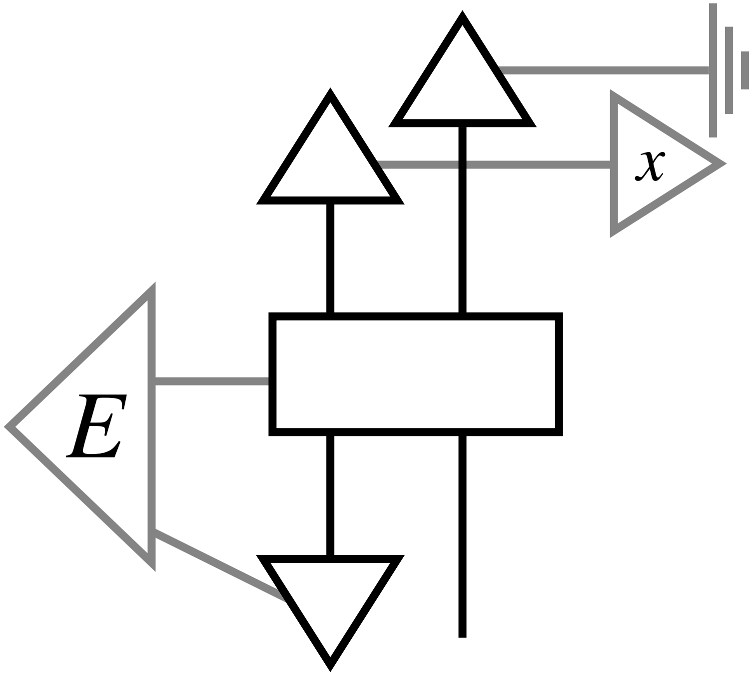

U_Φ
b


Φ

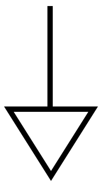
Operation



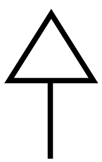
Operation



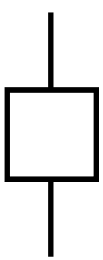
Generators



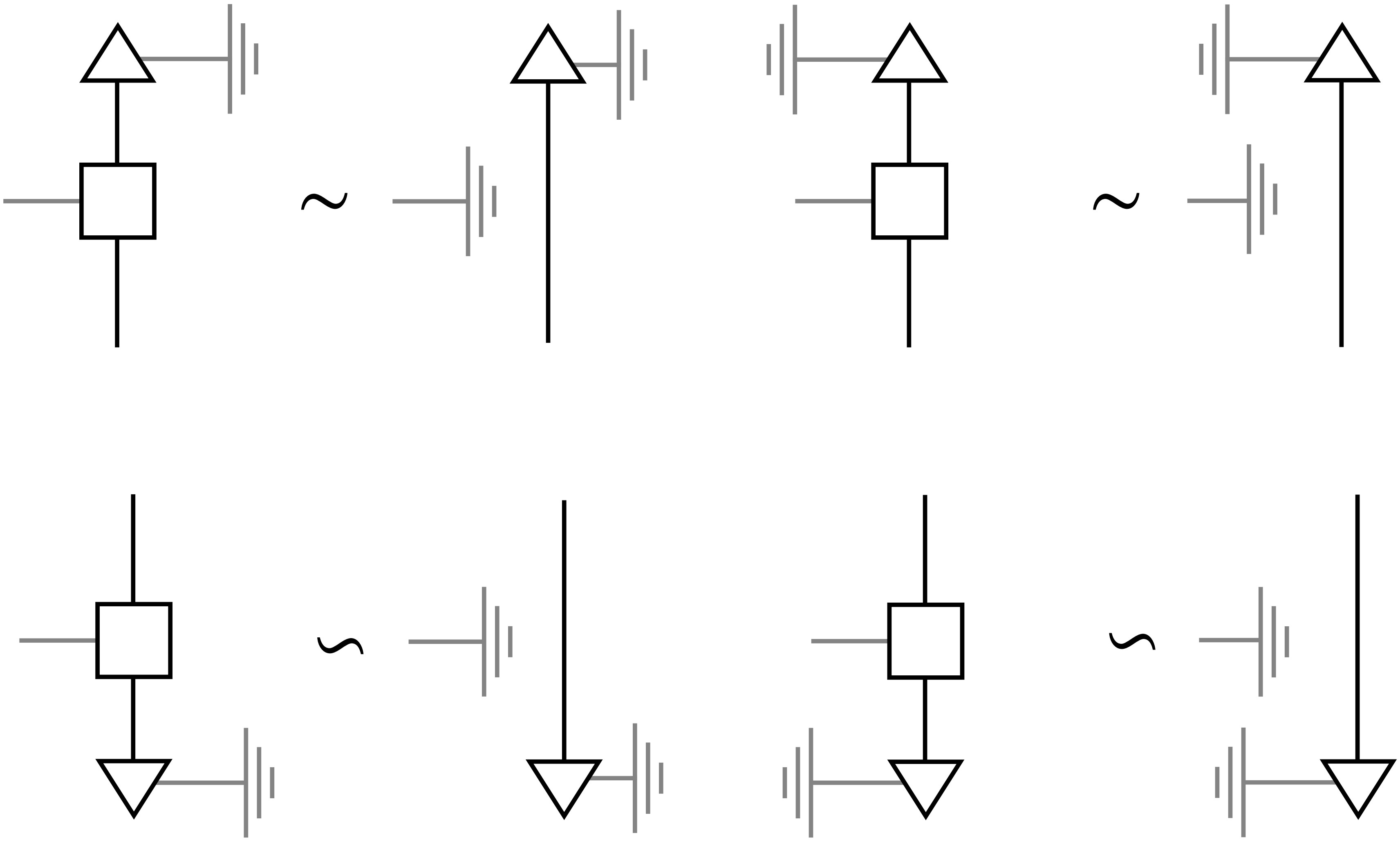
Pure state preparation



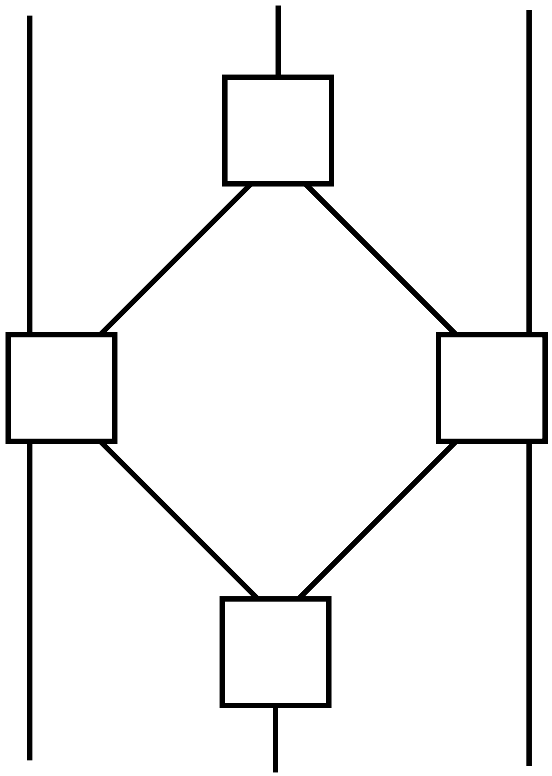
Pure state measurement

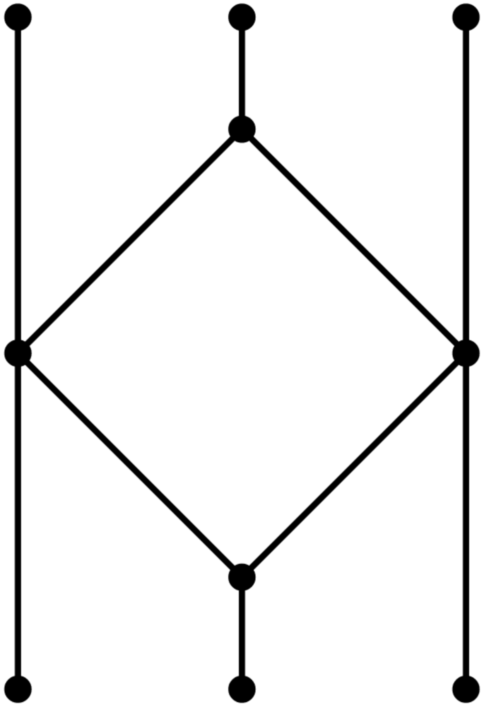


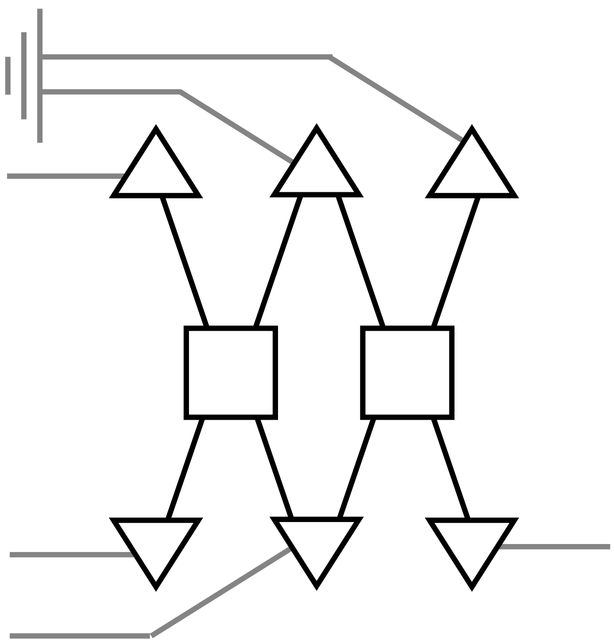
Unitary transformation



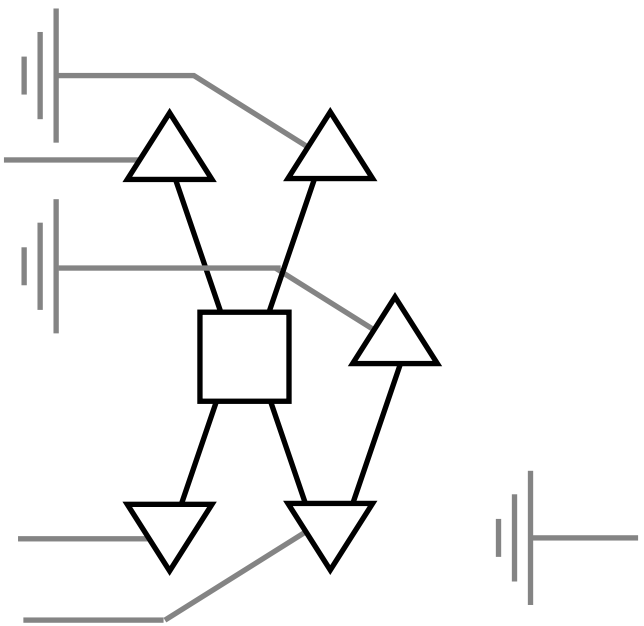
Compositionality

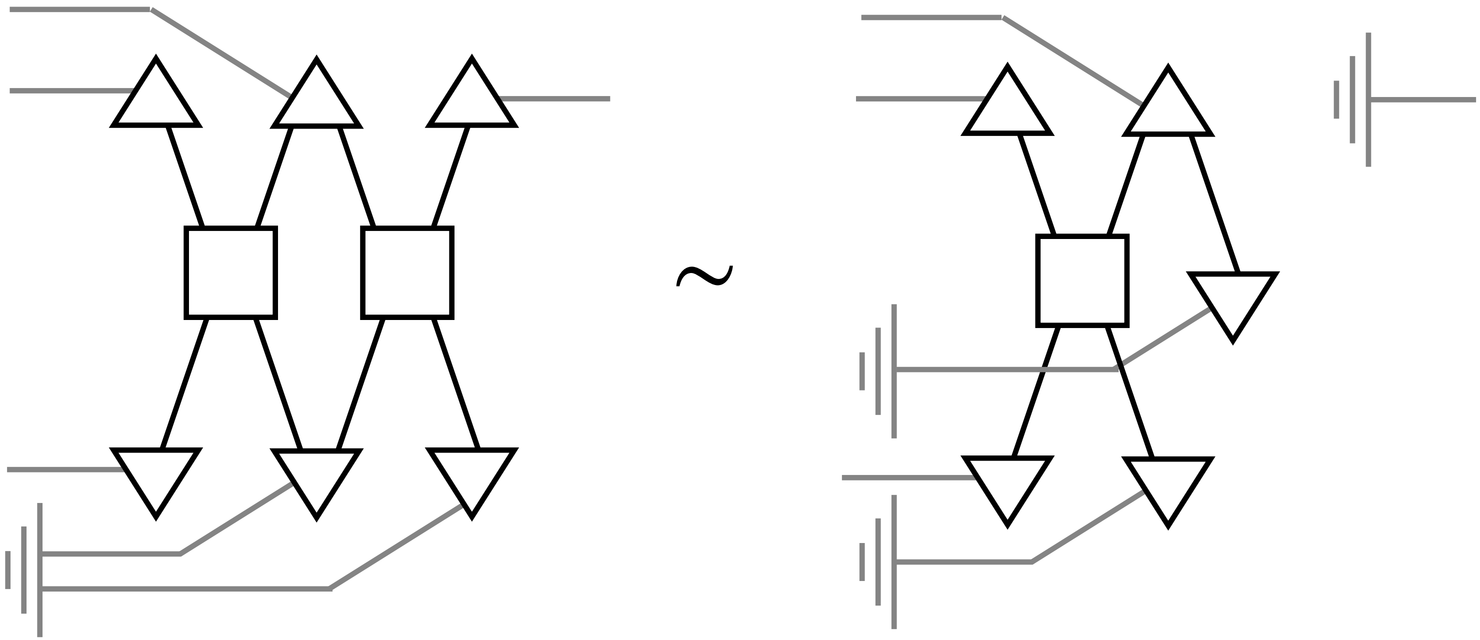






$\sim$





Summary

Separate causation (necessary correlations) from inference.

Think about causation time-symmetrically.

More goals:

Use Gibbs-preserving maps to talk about thermodynamical aspects.

Use a more elaborate quantum/classical interface.

Plan

- Quantum Information and the arrow of time
- Towards time-symmetric causation
- **Next steps**

Bell's theorems still bite.

Quantum [Un]Speakables II pp 119-142 | Cite as

Causarum Investigatio and the Two Bell's Theorems of John Bell

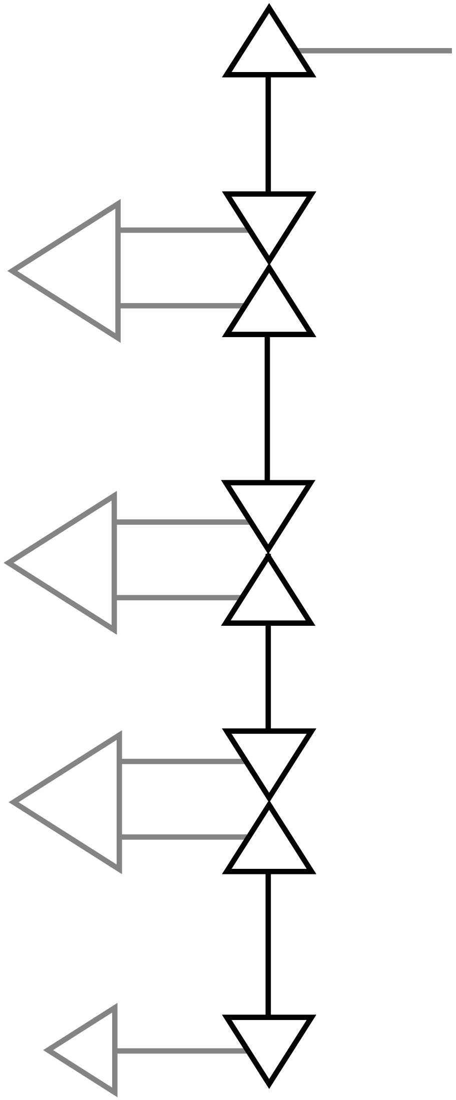
Authors Authors and affiliations

Howard M. Wiseman, Eric G. Cavalcanti

Modify:

- The causal structure: Compositionality ≠ Causality
- Inferential structure: Principle of Decorrelating Explanations

Time-symmetric reconstruction?



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Quantum theory from questions

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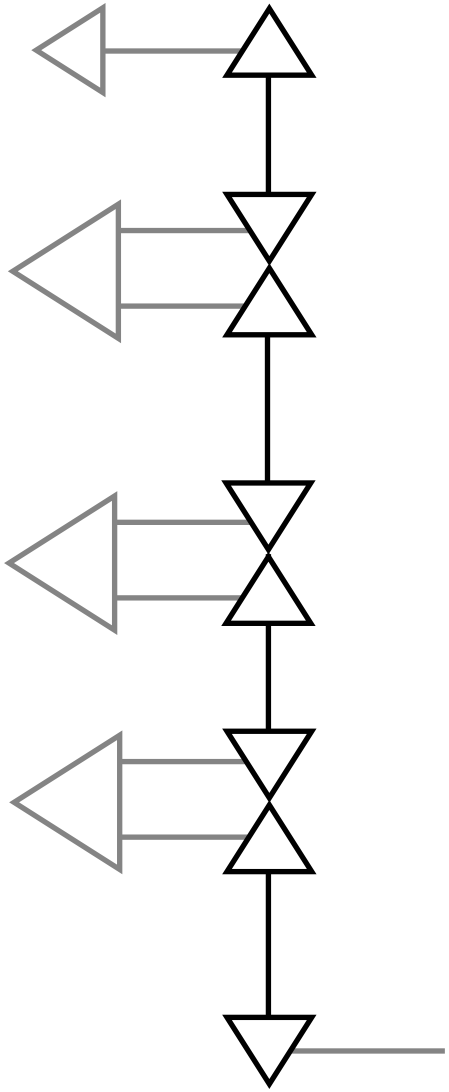
Phys. Rev. A **95**, 012102 – Published 3 January 2017

Replace "questions and answers" with "interactions and values of observables".

QM allows to calculate the probability of an event, given other events.

Time-symmetric reconstruction?



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Thank you

To be continued....

Thank you for listening!

Bell's theorems